



CARY ACADEMY

UPPER SCHOOL MATH CURRICULUM

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Upper School Curriculum Math Program Philosophy

The math teachers of Cary Academy seek to create a student experience that enriches and nurtures students' mathematical development. We accomplish this by

- o Routinely reflecting on our curriculum objectives to ensure the skills, concepts, and applications we teach are relevant and useful for students to navigate their world.
- o Vertically aligning our curriculum so that students will develop and build on their understandings in previous courses.
- o Developing course specific assessments that provide objective feedback to the student, teacher, and parent.
- o Collaborating within a subject to ensure all students are exposed to effective teaching practices that will deepen and enhance their understanding.
- o Seeking out best practices through professional development that requires reflection, sharing, and communicating with peers both inside and outside of Cary Academy.



Upper School Curriculum

Portrait of a Math Student

A student engaged in mathematics at Cary Academy is one who

- o Develops a mathematical library of skills, abilities, and resources and calls upon it when confronted with novel situations.
- o Engages in problem solving by taking risks as they explore problems, makes mistakes and learns from them, and persists until a solution is found.
- o Collaborates to develop, discuss, and deepen mathematical understandings.
- o Abstracts real world contexts into the language of mathematics.
- o Uses technology to purposefully and productively illuminate concepts while recognizing its limitations.



Upper School Curriculum

Math 100: Algebra

In this course, students begin by learning the basic rules, methods and concepts of algebra. They then explore linear equations, inequalities, polynomials, and quadratic equations. The development of both a symbolic and graphical understanding of the mathematics is emphasized. Students are also regularly asked to apply their understanding of the mathematics to real-world situations. Working on projects and learning to use technology appropriately are integral parts of the course.

GENERAL COURSE OBJECTIVES:

- o Students will be able to effectively use the order of operations and properties of algebra to simplify expressions and solve linear and quadratic equations.
- o Students will understand the relationship of slope to rates of change among two variables and analyze slope in context.
- o Students will become fluent in graphing functions on the coordinate plane and evaluating key characteristics such as domain, range, intercepts, and intersections.
- o Students will be able to take real world data, develop a function to model the data, and analyze it using the language of algebra.

SCOPE AND SEQUENCE:

UNIT	TARGETED SKILLS
The Language of Algebra	<ul style="list-style-type: none"> o apply the order of operations o simplify expressions o utilize properties (commutative, associative, distributive, etc.) o perform operations with fractions o set up and solve word problems
The Language of Algebra	<ul style="list-style-type: none"> o categorize real numbers (rational, irrational, integer, whole, natural) o perform operations with positive and negative real numbers o simplify square roots o find the absolute value o set up and solve word problems
Properties of Real Numbers	<ul style="list-style-type: none"> o solve 1, 2 and multi-step equations with variables on one side o solve for a variable in a formula o set up and solve proportions o solve percent problems o find percent change
Solving Linear Equations	<ul style="list-style-type: none"> o identify and plot points o represent relations and inverses o find domain and range from an equation, table, or graph o graph linear equations o evaluate and analyze arithmetic sequences
Graphing Relations and Functions	<ul style="list-style-type: none"> o calculate and interpret slope o write and graph direct variation o write and graph slope-intercept form o write and interpret point-slope form o interpret and convert standard form o analyze and write equations for parallel and perpendicular lines o construct and interpret linear regression models
Analyzing Linear Relationships	<ul style="list-style-type: none"> o solve systems of equation by graphing, substitution, and elimination o solving special linear systems and interpret their solutions o set up and solve linear systems word problems



Systems of Equations	<ul style="list-style-type: none">○ apply exponent properties○ convert between standard and scientific notation○ add, subtract and multiply polynomials○ factoring and solving polynomials
Exponents and Polynomials	<ul style="list-style-type: none">○ solve quadratic equations by factoring or the quadratic formula○ analyze the graph of a quadratic function○ graphing from vertex form
Quadratics	<ul style="list-style-type: none">○ simplify radical expressions○ perform operations with radicals○ solve radical equations○ set up and solve radical applications
Radicals	<ul style="list-style-type: none">○ set up and solve inequalities on a number line○ solve compound inequalities○ set up and solve open sentences with absolute value○ set up, graph, and solve linear inequalities on coordinate plane○ set up and solve linear inequality systems word problems
Inequalities	<ul style="list-style-type: none">○ apply the order of operations○ simplify expressions○ utilize properties (commutative, associative, distributive, etc.)○ perform operations with fractions○ set up and solve word problems



Upper School Curriculum

Math 200: Geometry

Students proceed through a Euclidean geometry course emphasizing deductive reasoning, sequential analysis, and proof. Logic plays a critical role in the development of properties for planar and spatial figures. Students also perform basic constructions and represent geometric figures in the Cartesian plane. Algebra is reinforced throughout the course. Students also learn to apply geometry to real-world problems..

GENERAL COURSE OBJECTIVES:

- o Students will be able to recognize and prove congruence and similarity of geometric shapes.
- o Students will be able to classify shapes by angles and sides, and analyze a shape’s area, perimeter, and the surface area and volume of 3 dimensional solids.
- o Students will develop a geometric vocabulary.
- o Students will be able to think logically through geometric proofs.

SCOPE AND SEQUENCE:

UNIT	TARGETED SKILLS
Points, Lines, Planes, and Angles	<ul style="list-style-type: none"> o Define terms commonly used in Geometry. o Use Postulates and Theorems relating points, lines, and planes. o Show how numbers are related to points on a line, and understand distance, length, congruence, and midpoint. o Understand rays and angles and the measurement of angles. o Use postulates to prove theorems.
Deductive Reasoning	<ul style="list-style-type: none"> o Study different forms of conditional statements that are often used in Geometry. o Review properties from Algebra that will be often used. o Begin the topic of what a proof is and how theorems are proved. o Apply the definitions and theorems about special pairs of angles. o Study and use definition and theorems about perpendicular lines. o Feel comfortable writing detailed two-column proofs. o Solve systems of equations from algebra and apply to Geometry.
Parallel lines and planes	<ul style="list-style-type: none"> o Distinguish between different line and plane relationships. o Identify the angles formed when two lines are cut by a transversal and the relationship of the angles. o Study different ways in which two lines can be parallel and be able to prove them parallel. o Classify triangles according to sides and angles and apply theorems about the sum of the measures of the angles in a triangle and the measure of the exterior angles. o Use inductive reasoning to generate formulas for finding the sum of the interior angles and exterior angles of an n-sided convex polygon. o Review writing the equation of a line from Algebra.
Congruent Triangles	<ul style="list-style-type: none"> o Identify corresponding parts of congruent figures and be able to prove triangles congruent using SSS, ASA, SAS, AAS, and HL. o Use congruent triangles to prove that the corresponding parts are congruent. o Apply theorems about isosceles triangles and use the definition and theorems in proofs. o Prove more than one pair of triangles congruent within the same proof. o Factor quadratics to solve a problem in Geometry.



Quadrilaterals	<ul style="list-style-type: none">○ Apply the definition of a parallelogram and the theorems about properties of a parallelogram.○ Prove that certain quadrilaterals are parallelograms.○ Prove useful theorems about parallel lines, including the mid-segment theorem for triangles.○ Determine when a parallelogram is a rectangle, rhombus, or a square.○ Identify properties of trapezoids and isosceles trapezoids.
Coordinate Geometry	<ul style="list-style-type: none">○ Use the slope, midpoint, and distance formula to prove the best name for a quadrilateral given the coordinates of its 4 vertices.
Inequalities in Geometry	<ul style="list-style-type: none">○ Apply the inequality theorems and corollaries for one triangle and for two triangles.○ Find the possible side lengths for the third side of a triangle given the other two side lengths.
Similar Polygons	<ul style="list-style-type: none">○ Review ratios, proportions, and simplifying fractions.○ Determine when proportions are equivalent.○ Determine whether two polygons are similar and find a scale factor.○ Use similar triangles to deduce information about segments or angles.○ Apply postulates and theorems about similar triangles.
Right Triangles	<ul style="list-style-type: none">○ Determine the geometric mean between two numbers.○ State and apply the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle.○ State and apply the Pythagorean Theorem.○ Apply the converse of the Pythagorean Theorem to determine if a triangle is right, acute, or obtuse.○ Determine the lengths of two sides of a 45-45-90 or a 30-60-90 triangle when the third length is known.○ Define the tangent, sine, and cosine ratios for a right triangle.○ Solve right triangle problems by correct selection and use of the trig ratios.
Circles	<ul style="list-style-type: none">○ Define a circle, sphere, and terms related to them.○ Recognize circumscribed and inscribed polygons and circles.○ Apply theorems that relate tangents and radii.○ Define and apply properties of arcs and central angles.○ Apply theorems about the chords of a circle.○ Solve problems and prove statements involving inscribed angles, chords, secants, and tangents.○ Solve problems involving lengths of chords, secant segments, and tangent segments.
Areas of Plane Figures	<ul style="list-style-type: none">○ Understand and apply the formulas for areas of quadrilaterals and regular polygons.○ Understand and apply the formulas for circumference and area of circles.○ Use the formulas for arc lengths and the areas of sectors of a circle.○ Find the ratio of the areas of two triangles and apply the relationships between scale factors, perimeters, and areas of similar figures.
Areas and Volumes of Solids	<ul style="list-style-type: none">○ Identify the parts of prisms, pyramids, cylinders, and cones.○ Find the lateral areas, total areas, and volumes of right prisms, regular pyramids, right cylinders, and right cones.○ Find the area and volume of a sphere.
Transformations	<ul style="list-style-type: none">○ Understand translations, reflections, and rotations.○ Recognize the properties of basic mappings.



Upper School Curriculum

Math 250: Honors Geometry

This honors course will allow students to explore Euclidean geometry with an emphasis on deductive reasoning, sequential analysis, and proof. Logic plays a critical role in the development of properties for planar and spatial figures. Students also perform basic constructions and represent geometric figures in the Cartesian plane. Algebra is reinforced throughout the course. Students also learn to apply geometry to real-world problems.

GENERAL COURSE OBJECTIVES

- Students will be able to recognize and prove congruence and similarity of geometric shapes.
- Students will be able to classify shapes by angles and sides, and analyze a shape's area, perimeter, and the surface area and volume of 3 dimensional solids.
- Students will develop a geometric vocabulary.
- Students will be able to think logically through geometric proofs.

SCOPE AND SEQUENCE

UNIT	TARGETED SKILLS
Points, Lines, Planes, and Angles	<ul style="list-style-type: none">○ Define terms commonly used in Geometry.○ Use Postulates and Theorems relating points, lines, and planes.○ Show how numbers are related to points on a line, and understand distance, length, congruence, and midpoint.○ Understand rays and angles and the measurement of angles.○ Use postulates to prove theorems.
Deductive Reasoning	<ul style="list-style-type: none">○ Study different forms of conditional statements that are often used in Geometry.○ Review properties from Algebra that will be often used.○ Begin the topic of what a proof is and how theorems are proved.○ Apply the definitions and theorems about special pairs of angles.○ Study and use definition and theorems about perpendicular lines.○ Feel comfortable writing detailed two-column proofs.○ Solve systems of equations from algebra and apply to Geometry.
Parallel lines and planes	<ul style="list-style-type: none">○ Distinguish between different line and plane relationships.○ Identify the angles formed when two lines are cut by a transversal and the relationship of the angles.○ Study different ways in which two lines can be parallel and be able to prove them parallel.○ Classify triangles according to sides and angles and apply theorems about the sum of the measures of the angles in a triangle and the measure of the exterior angles.○ Use inductive reasoning to generate formulas for finding the sum of the interior angles and exterior angles of an n-sided convex polygon.○ Review writing the equation of a line from Algebra.



Congruent Triangles	<ul style="list-style-type: none">o Identify corresponding parts of congruent figures and be able to prove triangles congruent using SSS, ASA, SAS, AAS, and HL.o Use congruent triangles to prove that the corresponding parts are congruent.o Apply theorems about isosceles triangles and use the definition and theorems in proofs.o Prove more than one pair of triangles congruent within the same proof.o Factor quadratics to solve a problem in Geometry.
Quadrilaterals	<ul style="list-style-type: none">o Apply the definition of a parallelogram and the theorems about properties of a parallelogram.o Prove that certain quadrilaterals are parallelograms.o Prove useful theorems about parallel lines, including the mid-segment theorem for triangles.o Determine when a parallelogram is a rectangle, rhombus, or a square.o Identify properties of trapezoids and isosceles trapezoids.
Coordinate Geometry	<ul style="list-style-type: none">o Use the slope, midpoint, and distance formula to prove the best name for a quadrilateral given the coordinates of its 4 vertices.
Inequalities in Geometry	<ul style="list-style-type: none">o Apply the inequality theorems and corollaries for one triangle and for two triangles.o Find the possible side lengths for the third side of a triangle given the other two side lengths.
Similar Polygons	<ul style="list-style-type: none">o Review ratios, proportions, and simplifying fractions.o Determine when proportions are equivalent.o Determine whether two polygons are similar and find a scale factor.o Use similar triangles to deduce information about segments or angles.o Apply postulates and theorems about similar triangles.
Right Triangles	<ul style="list-style-type: none">o Determine the geometric mean between two numbers.o State and apply the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle.o State and apply the Pythagorean Theorem.o Apply the converse of the Pythagorean Theorem to determine if a triangle is right, acute, or obtuse.o Determine the lengths of two sides of a 45-45-90 or a 30-60-90 triangle when the third length is known.o Define the tangent, sine, and cosine ratios for a right triangle.o Solve right triangle problems by correct selection and use of the trig ratios.
Circles	<ul style="list-style-type: none">o Define a circle, sphere, and terms related to them.o Recognize circumscribed and inscribed polygons and circles.o Apply theorems that relate tangents and radii.o Define and apply properties of arcs and central angles.o Apply theorems about the chords of a circle.o Solve problems and prove statements involving inscribed angles, chords, secants, and tangents.o Solve problems involving lengths of chords, secant segments, and tangent segments.
Areas of Plane Figures	<ul style="list-style-type: none">o Understand and apply the formulas for areas of quadrilaterals and regular polygons.o Understand and apply the formulas for circumference and area of circles.o Use the formulas for arc lengths and the areas of sectors of a circle.o Find the ratio of the areas of two triangles and apply the relationships between scale factors, perimeters, and areas of similar figures.
Areas and Volumes of Solids	<ul style="list-style-type: none">o Identify the parts of prisms, pyramids, cylinders, and cones.o Find the lateral areas, total areas, and volumes of right prisms, regular pyramids, right cylinders, and right cones.o Find the area and volume of a sphere.
Transformations	<ul style="list-style-type: none">o Understand translations, reflections, and rotations.o Recognize the properties of basic mappings.



Upper School Curriculum

Math 300: Algebra II

Students will develop proficiency with algebraic expressions and equations to support their study of functions. Linear, absolute value, piecewise, quadratic and polynomial functions, as well as systems of equations and matrices are the focus of this course. Rational expressions, equations and functions round out the year. Each of these topics lays the foundation for further studies in mathematics, including an emphasis on their applications.

GENERAL COURSE OBJECTIVES:

- o Students will be able to apply the order of operations and properties of algebra to simplify expressions and solve linear, quadratic, absolute value, piece-wise, polynomial, rational, exponential, and logarithmic equations and inequalities.
- o Students will be able to analyze real world data, develop a function to model the data and interpret it using the language of algebra.
- o Students will be able to use matrices as a tool for organizing data and solving problems involving systems of equations.
- o Students will be able to relate graphical and algebraic representations of functions and sketch the graphs of higher-order functions without the aid of a calculator.

SCOPE AND SEQUENCE:

UNIT	TARGETED SKILLS
Equations and Inequalities	<ul style="list-style-type: none"> o Rewrite formulas and equations o Solve linear inequalities o Solve absolute value equations o Solve absolute value inequalities
Linear Equations and Functions	<ul style="list-style-type: none"> o Represent relations and functions o Find slope and rate of change o Graph and write equations of lines o Find the line of best fit o Use absolute value functions and transformations o Graph linear inequalities in two variables o Model direct variation o Graph and work with piecewise functions
Linear Systems and Matrices	<ul style="list-style-type: none"> o Use notation to describe the elements and dimensions of a matrix o Perform matrix operations: addition, subtraction, and multiplication o Find the determinant of a 2-by-2 matrix
Quadratic Functions	<ul style="list-style-type: none"> o Graph using the three forms o Solve by factoring o Solving with radicals o Solve by completing the square o Find the center of a circle by completing the square o Complex numbers and their operations o Use the quadratic formula and discriminant o Graph and solve quadratic inequalities o Apply quadratics



Polynomials	<ul style="list-style-type: none">○ Use properties of exponents○ Evaluate, graph and analyze polynomials○ Perform operations on polynomials○ Factor and solve polynomials○ Apply the remainder and factor theorems○ Find rational zeros○ Apply the fundamental theorem of algebra○ Write polynomial functions and models
Rational Exponents and Radical Functions	<ul style="list-style-type: none">○ Use and apply properties of rational exponents○ Convert between rational and radical form○ Perform composition○ Use inverse functions○ Graph and solve radicals
Exponential and Log Functions	<ul style="list-style-type: none">○ Graph exponential growth and decay functions○ Use functions involving e○ Evaluate, apply properties of and graph logarithms and exponentials
Rational Functions	<ul style="list-style-type: none">○ Graph rational functions○ Perform operations on rational expressions○ Solve rational equations



Upper School Curriculum

Math 350: Honors Algebra II

Students will develop proficiency with algebraic expressions including linear, quadratic, rational, radical, exponential, logarithmic, and polynomial functions. These are used as tools for understanding real-world applications of mathematics, so word problems involving applications of these models will also be emphasized. Additionally, given the honors level of the course, topics of interest will be explored in significant detail.

GENERAL COURSE OBJECTIVES

- Students will be able to apply the order of operations and properties of algebra to simplify expressions and solve linear, quadratic, absolute value, piece-wise, polynomial, rational, logarithmic and exponential equations and inequalities.
- Students will be able to analyze real world data, develop a function to model the data and interpret it using the language of algebra.
- Students will be able to use matrices as a tool for organizing data and solving problems involving systems of equations.
- Students will be able to relate graphical and algebraic representations of functions and sketch the graphs of higher-order functions without the aid of a calculator.

SCOPE AND SEQUENCE:

UNIT	TARGETED SKILLS
Equations and Inequalities	<ul style="list-style-type: none"> ○ Rewrite formulas and equations ○ Solve linear inequalities ○ Solve absolute value equations ○ Solve absolute value inequalities
Linear Equations and Functions	<ul style="list-style-type: none"> ○ Represent relations and functions ○ Find slope and rate of change ○ Graph and write equations of lines ○ Find the line of best fit ○ Use absolute value functions and transformations ○ Graph linear inequalities in two variables ○ Model direct variation ○ Graph and work with piecewise functions
Linear Systems and Matrices	<ul style="list-style-type: none"> ○ Solving linear systems with two and three variables with multiple methods ○ Perform basic matrix operations ○ Find the determinant of a matrix ○ Use matrices to solve linear systems
Quadratic Functions	<ul style="list-style-type: none"> ○ Graph using the three forms ○ Solve by factoring ○ Solving with radicals ○ Solve by completing the square ○ Complex numbers and their operations ○ Use the quadratic formula and discriminant ○ Graph and solve quadratic inequalities ○ Apply quadratics



Polynomials	<ul style="list-style-type: none">○ Use properties of exponents○ Evaluate, graph and analyze polynomials○ Perform operations on polynomials○ Factor and solve polynomials○ Apply the remainder and factor theorems○ Find rational zeros○ Apply the fundamental theorem of algebra○ Write polynomial functions and models
Rational Exponents and Radical Functions	<ul style="list-style-type: none">○ Use and apply properties of rational exponents○ Convert between rational and radical form○ Perform composition○ Use inverse functions○ Graph and solve radicals
Exponential and Log Functions	<ul style="list-style-type: none">○ Graph exponential growth and decay functions○ Use functions involving e○ Evaluate, apply properties of and graph logarithms and exponentials
Rational Functions	<ul style="list-style-type: none">○ Graph rational functions○ Perform operations on rational expressions○ Solve rational equations
Conics	<ul style="list-style-type: none">○ Graph and write equations for basic conics○ Develop appropriate conic terminology



Upper School Curriculum

Math 380: Topics in Probability/Combinatorics Number Theory & Geometry

This course is designed to help students interested in high school mathematics competitions to learn the theory behind solving problems which the standard mathematical curriculum does not adequately address. Students will be exposed to large-group discussions about topics such as probability, combinatorics and number theory. The fundamentals behind these topics, though, are largely intuitive and are accessible to students of all ages. Other topics, such as Geometry, will be examined with the specific aim of gathering tools and techniques that are needed to solve the most challenging math competition problems. A strong emphasis will be placed on students working collaboratively to solve fewer, more difficult problems within an extended time frame, and on students learning from each other through student-led presentation of solutions. May be repeated for credit.

GENERAL COURSE OBJECTIVES

- Students will be able to use the problem solving tools and techniques they have learned to make progress in approaching types of probability, number theory and geometry problems they have not previously encountered.
- Students will understand the value of personal goal setting and focusing on improvement, rather than simply assessing themselves in comparison to their classmates.
- Students will appreciate that exploring familiar mathematical topics in greater detail helps to solidify foundational principles and connections between math coursework from different years.
- Students will appreciate that understanding core ideas can reduce their dependence on memorization and make them more successful independent learners when exposed to new math topics in the future.

SCOPE AND SEQUENCE

UNIT	TARGETED SKILLS
Factoring and Rules of Exponents	<ul style="list-style-type: none"> ○ Apply basic exponent and factoring rules to solve advanced problems.
Problem Solving	<ul style="list-style-type: none"> ○ Apply basic problem solving techniques such as solving a number of smaller cases of a given problem and searching for a pattern. ○ Make progress toward solutions of problems, even when they seem at first to be of an unfamiliar type.
"Wordy" Problems	<ul style="list-style-type: none"> ○ Interpret word problems involving complex, often confusing phrasing to decipher meaning and reduce the problem to a solvable form.
Number Theory	<ul style="list-style-type: none"> ○ Recognize when a problem's solutions involves only integer possibilities, and that knowing that makes the problem solvable.
Geometry	<ul style="list-style-type: none"> ○ Thoroughly understand the foundational problem solving techniques of Pythagorean Theorem and similar triangles. ○ Use basic principles and relationships to derive more complex techniques related to triangle and circle geometry.



Probability	<ul style="list-style-type: none">○ Thoroughly understand the foundational principles of combinatorics: counting arrangements, combinations and permutations.○ Understand the connection between combinatorics and probability, using an appropriately chosen sample space.○ Apply basic principles of probability to solve varied, complex application-level problems
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Upper School Curriculum

Math 380: Topics in Probability/Combinatorics Number Theory & Geometry

Students will expand their understanding of functions to include trigonometric, rational, logarithmic and exponential functions. Additional topics covered include sequences and series, graphical transformations, trigonometry and probability. The fundamentals of Algebra continue to be integrated thorough out the course with an emphasis on application.

GENERAL COURSE OBJECTIVES

- Students will be able to analyze a library of functions in terms of the fundamental toolkit attributes such as domain, range, intercepts, and limits.
- Students will be able to use functions learned to model real world data and draw conclusions.
- Students will be able to use the algebraic form of a function to create an accurate sketch without the aid of a calculator.
- Students will be able to use trigonometry to simplify expressions, solve equations, prove identities, and model real world data.
- Students will be able to recognize the patterns within sequences and series and use arithmetic and geometric series to analyze and model real world data.
- Students will be able to develop a fluency with the mathematics to aid in general problem solving in science and math.
- Students will be able to implement basic principles of probability to investigate statistical situations

SCOPE AND SEQUENCE

UNIT	TARGETED SKILLS
Toolkit Functions	<ul style="list-style-type: none"> ○ Identify basic functions & their characteristics ○ represent domain and range using proper notation ○ understand the form and features of all transformations
Polynomial Functions	<ul style="list-style-type: none"> ○ Sketch polynomials ○ Identify end behavior, multiplicity, degree, symmetry and intercepts ○ Determine a function given its roots ○ Synthetic & long division ○ Factor polynomials ○ Evaluate domain and range ○
Rational Functions	<ul style="list-style-type: none"> ○ Sketch a rational function ○ Identify intercepts, holes, asymptotes, domain and range ○ Evaluate limits graphically and algebraically ○ Evaluate domain and range



Piecewise Functions	<ul style="list-style-type: none">○ Sketch piecewise functions○ Evaluate limits graphically and algebraically○ Evaluate domain and range○ Evaluate limits graphically and algebraically○
Exponential and Log Functions	<ul style="list-style-type: none">○ Sketch exponential, log, logistic & rational power functions○ Apply transformations and identify critical points & asymptotes○ Solve exponential and logarithmic equations○ Use properties of logs○ Model exponential, log and logistic applications○ Evaluate domain and range○ Evaluate limits graphically and algebraically
Triangle Trigonometry	<ul style="list-style-type: none">○ Solve the sides and angles of any triangle○ Laws of sines and cosines○ Calculate area of any triangle
Circular Trigonometry	<ul style="list-style-type: none">○ Understand the relationship between radians and degrees○ Fill in a Unit Circle○ Calculate six trig functions of any angle○ Identify co-terminal & reference angles○ Solve trig & inverse trig equations
Trigonometric Functions	<ul style="list-style-type: none">○ Graph the six sinusoidal functions○ Identify period, amplitude, phase & vertical shifts○ Model trigonometric applications○ Evaluate domain and range○ Evaluate limits graphically and algebraically○
Analytic Trigonometry	<ul style="list-style-type: none">○ Evaluate trig identities○ Solve trigonometric equations
Sequences and Series	<ul style="list-style-type: none">○ Understand the nature of a sequence and identify the pattern○ Analyze arithmetic sequences & series○ Analyze geometric sequences & series○ Use sigma notation
Probability	<ul style="list-style-type: none">○ Experimental vs. Theoretical probability○ Independent, dependent and compound events○ Evaluate conditional probability○ Complete two-way frequency tables



Upper School Curriculum

Math 435: Functions and Modeling

Functions and Modeling exposes students to the concepts and skills that they will encounter in a college math course. With a heavy emphasis on collaboration, students will work to enhance their algebraic understanding of essential concepts necessary for success in college. They will complete projects that are relevant to their interests and illustrate the applications of mathematics in the real world.

GENERAL COURSE OBJECTIVES:

- Students will enhance their understanding of algebraic functions and operations.
- Students will develop an understanding of probability and statistics concepts
- Students will explore trigonometric concepts
- Students will prepare for college level mathematics

SCOPE AND SEQUENCE:

UNIT	TARGETED SKILLS
College Math	<ul style="list-style-type: none">○ Review of the toolkit functions, transformations, and analysis of critical features of functions.○ Solving linear, quadratic, polynomial, rational, exponential, and logarithmic equations and inequalities.○ Work with parametric functions to model real world data.
Analytic Trigonometry	<ul style="list-style-type: none">○ Solve right triangles○ Solve non-right triangles○ Understand and use the relationship between radian and degree measures.○ Understand and use the relationships within the Unit Circle to evaluate angle measures, develop identities, and solve equations.
Trig Functions	<ul style="list-style-type: none">○ Graph the sinusoidal functions and be able to identify period, amplitude, phase shifts, vertical shifts.○ Model real world data using trigonometric functions.
Probability	<ul style="list-style-type: none">○ Be able to use set notation to describe events.○ Be able to identify Independent and dependent events and calculate probabilities.○ Use two-way tables to calculate probabilities of individual events, compound events, and Mutually exclusive events.○ Experimental and theoretical probabilities
Statistics	<ul style="list-style-type: none">○ Be able to collect, summarize and visualize univariate data using.○ Recognize situations involving binomial random variables and be able to perform calculations.○ Use the normal distribution to calculate probabilities.○ Make inferences using probability distributions.○ Construct linear models using regression methods.



Upper School Curriculum

Math 440: Precalculus

Students will expand their understanding of functions to include trigonometric, rational, logarithmic and exponential functions. Additional topics covered include sequences and series, graphical transformations, trigonometry and probability. The fundamentals of Algebra continue to be integrated thorough out the course with an emphasis on application.

GENERAL COURSE OBJECTIVES:

- Students will be able to analyze a library of functions in terms of the fundamental toolkit attributes such as domain, range, intercepts, and limits.
- Students will be able to use functions learned to model real world data and draw conclusions.
- Students will be able to use the algebraic form of a function to create an accurate sketch without the aid of a calculator.
- Students will be able to use trigonometry to simplify expressions, solve equations, prove identities, and model real world data.
- Students will be able to recognize the patterns within sequences and series and use arithmetic and geometric series to analyze and model real world data.
- Students will be able to develop a fluency with the mathematics to aid in general problem solving in science and math.
- Students will be able to implement basic principles of probability to investigate statistical situations.

SCOPE AND SEQUENCE:

UNIT	TARGETED SKILLS
Toolkit Functions	<ul style="list-style-type: none">○ Identify basic functions & their characteristics○ Represent domain and range using proper notation○ Understand the form and features of all transformations
Polynomial Functions	<ul style="list-style-type: none">○ Sketch polynomials○ Identify end behavior, multiplicity, degree, symmetry and intercepts○ Determine a function given its roots○ Synthetic & long division○ Factor polynomials○ Evaluate domain and range
Rational Functions	<ul style="list-style-type: none">○ Sketch a rational function○ Identify intercepts, holes, asymptotes, domain and range○ Evaluate limits graphically and algebraically○ Evaluate domain and range
Piecewise Functions	<ul style="list-style-type: none">○ Sketch piecewise functions○ Evaluate limits graphically and algebraically○ Evaluate domain and range○ Evaluate limits graphically and algebraically



Exponential and Log Functions	<ul style="list-style-type: none">○ Sketch exponential, log, logistic & rational power functions○ Apply transformations and identify critical points & asymptotes○ Solve exponential and logarithmic equations○ Use properties of logs○ Model exponential, log and logistic applications○ Evaluate domain and range○ Evaluate limits graphically and algebraically
Triangle Trigonometry	<ul style="list-style-type: none">○ Solve the sides and angles of any triangle○ Laws of sines and cosines○ Calculate area of any triangle
Circular Trigonometry	<ul style="list-style-type: none">○ Understand the relationship between radians and degrees○ Fill in a Unit Circle○ Calculate six trig functions of any angle○ Identify co-terminal & reference angles○ Solve trig & inverse trig equations
Trigonometric Functions	<ul style="list-style-type: none">○ Graph the six sinusoidal functions○ Identify period, amplitude, phase & vertical shifts○ Model trigonometric applications○ Evaluate domain and range○ Evaluate limits graphically and algebraically
Analytic Trigonometry	<ul style="list-style-type: none">○ Evaluate trig identities○ Solve trigonometric equations
Sequences and Series	<ul style="list-style-type: none">○ Understand the nature of a sequence and identify the pattern○ Analyze arithmetic sequences & series○ Analyze geometric sequences & series○ Use sigma notation
Probability	<ul style="list-style-type: none">○ Experimental vs. Theoretical probability○ Independent, dependent and compound events○ Evaluate conditional probability○ Complete two-way frequency tables



Upper School Curriculum

Math 450: Honors Precalculus

This honors course is designed to prepare students for the Calculus and other collegiate level mathematics and science courses. As a result, the primary focus of Honors Precalculus (H) is to bring together critical skills and concepts from algebra and geometry and to integrate the language and concepts of functions. Fundamental to the study of higher mathematics is a thorough understanding of the properties of exponential, logarithmic, polynomial, rational, and trigonometric functions. Students develop this understanding while investigating applications, which include vectors, polar coordinates, recursion, analytic geometry, probability, and combinatorics.

GENERAL COURSE OBJECTIVES:

- Students will be able to analyze a library of functions in terms of the fundamental toolkit attributes such as domain, range, intercepts, continuity, symmetry, asymptotes and limits.
- Students will be able to use functions learned to model real world data and draw conclusions.
- Students will be able to use the algebraic form of a function to create an accurate sketch with and without the aid of a calculator.
- Students will be able to use trigonometry to simplify expressions, solve equations, prove identities, and model real world data.
- Students will be able to recognize the patterns within sequences and series and use arithmetic and geometric series to analyze and model real world data.
- Students will develop a fluency with the mathematics utilized for general problem solving in science and math.
- Students will develop skills in preparation for AP Calculus.

SCOPE AND SEQUENCE:

UNIT	TARGETED SKILLS
Algebra Review	<ul style="list-style-type: none"> ○ Factor any polynomial ○ Identify the domain of a function ○ Solve linear, quadratic and rational equations ○ Solve literal equations ○ Compute with complex numbers
Toolkit Functions	<ul style="list-style-type: none"> ○ For any toolkit function (Linear, Absolute Value, Quadratic, Polynomial, Rational, Exponential, Logarithmic, and Piece-wise) be able to: ○ Identify parent functions ○ Identify the critical features of any function ○ Understand the effects of transformations on the toolkit functions ○ Perform operations with functions ○ Use technology to analyze any function
Polynomials	<ul style="list-style-type: none"> ○ Sketch polynomial functions by identifying end behaviors, multiplicity of roots and intercepts ○ Determine a function given its roots ○ Solve polynomial equations and inequalities
Rational Functions	<ul style="list-style-type: none"> ○ Sketch the graph of a rational function using intercepts, holes, asymptotes, domain and range ○ Evaluate limits graphically ○ Solve rational equations and inequalities



Exponential and Log Functions	<ul style="list-style-type: none">○ Sketch exponential functions○ Apply transformations and identify critical points and asymptotes○ Solve exponential equations○ Understand the relationship between domain and range of inverse functions and relations○ Sketch log functions○ Apply transformations and identify critical points and asymptotes○ Use properties of logs○ Solve log equations and inequalities○ Model with exponential and log functions
Trigonometry	<ul style="list-style-type: none">○ Understand the relationship between radian and degree measures○ Understand the relationships within the unit circle○ Evaluate angles using standard reference angles
Trigonometric Functions	<ul style="list-style-type: none">○ Graph the six sinusoidal functions and their reciprocals○ Sketch sinusoidal functions○ Identify period, amplitude, phase shifts, vertical shifts○ Use properties of trig functions to evaluate even and odd properties and periodic properties○ Model with trigonometric functions
Analytic Trigonometry	<ul style="list-style-type: none">○ Establish trig identities○ Solve trig equations○ Evaluate inverse trig functions○ Solve any triangle
Parametrics	<ul style="list-style-type: none">○ Convert expressions into polar form and back into rectangular form○ Evaluate and sketch parametric functions○ Model using parametric functions
Polar Equations	<ul style="list-style-type: none">○ Convert expressions and equations to polar form and back into rectangular form○ Graph polar functions and understand how the coefficients can impact the graphs○ Evaluate powers and roots of complex numbers
Conics	<ul style="list-style-type: none">○ Sketch the graph of conic sections by identifying critical points○ Model using conic sections○ Solve non-linear systems of equations
Sequences and Series	<ul style="list-style-type: none">○ Understand the nature of a sequence and identify the pattern○ Work with arithmetic sequences○ Work with geometric sequences○ Use sigma notation○ Calculate arithmetic series○ Calculate geometric series



Upper School Curriculum

Math 510: Calculus

This course is an introduction to the methods and ideas of the Calculus. Students will utilize skills developed earlier in their mathematical courses to address conceptual and applied topics related to the Calculus. Some of the topics that may be covered are limits, derivatives, anti-derivatives, optimization, and related rates.

GENERAL COURSE OBJECTIVES:

- Students will review the analysis and sketching details of the fundamental functions from Precalculus.
- Students will review all algebraic techniques used to solve equations from Precalculus and Algebra II.
- Students will understand the concept of a limit as it applies to functions they have studied in their previous courses.
- Students will understand the definition of a derivative and be able to use the limit definition to find derivatives.
- Students will be able to evaluate derivatives and recognize their connection to slope and real world applications.
- Students will understand implicit differentiation and be able to use it to solve derivatives.
- Students will understand the geometric interpretation of an integral.
- Students will be able to understand and evaluate definite and indefinite integrals.
- Students will be able to understand applications of integration dealing with area, volumes, and arc length.

SCOPE AND SEQUENCE:

UNIT	TARGETED SKILLS
Algebra Review	<ul style="list-style-type: none"> ○ Solve linear, quadratic, radical, rational, and absolute value equations. ○ Solve linear, quadratic, radical, rational, and absolute value inequalities. ○ Identify domains of any function. ○ Analyze lines and circles in terms of slope, midpoints, distance, and tangents. ○ Sketch and analyze the 12 fundamental functions.
Limits	<ul style="list-style-type: none"> ○ Work with limit notation ○ Analyze limits from a specific direction, both directions, and towards infinity. ○ Use direct substitution to evaluate limits. ○ Use sketching and table to evaluate limits. ○ Apply rules for limits to evaluate limits.
Derivatives	<ul style="list-style-type: none"> ○ Understand and apply the limit definition of a derivative. ○ Understand how rate of change relates to a derivative. ○ Understand the relationship between the graph of a function and its derivatives graph. ○ Apply the limit definition to determine derivatives.
Differentiation	<ul style="list-style-type: none"> ○ Find the derivatives of the following functions: Polynomial, exponential, logarithmic, and trigonometric. ○ Apply the product and quotient rules to find derivatives. ○ Apply the chain rule. ○ Perform implicit differentiation.



Applications of Differentiation	<ul style="list-style-type: none">○ Evaluate situations requiring maximum and minimum values.○ Evaluate situations requiring related rates.○ Apply L'Hopital's rule.○ Evaluate situations that require optimization.
Integrals	<ul style="list-style-type: none">○ Understand the definition of an integral in terms of areas and distances.○ Evaluate a definite integral.○ Use the fundamental theorem of calculus.○ Use substitution to evaluate integrals.○ Use the technique of integration by parts.○ Apply addition techniques to evaluate integrals.○ Approximate improper integrals.
Applications of integrals	<ul style="list-style-type: none">○ Apply integral techniques to applications involving areas, volumes, arc lengths, and average velocities.
Sequences and Series	<ul style="list-style-type: none">○ Understand sigma notation.○ Evaluate expressions involving sigma notation.○ Work with properties of sequences and series as a precursor to Taylor and MacLauren series.



Upper School Curriculum

Math 525: Advanced Statistical Theory and Applications

Advanced Statistical Theory and Applications is a calculus based course that takes advantage of the latest research and application of analytics in the academic and business worlds. Using JMP® software, graphing calculator, and online tools students will examine the four core principles of the course: Exploring data, sampling and experimentation, anticipating patterns and statistical inference. Through the application of Calculus this course will provide a theoretical approach to the concepts. Most students choose to sit for the Advanced Placement (AP) examination.

GENERAL COURSE OBJECTIVES:

- o Students will be able to design and evaluate experiments and sampling procedures to collect data.
- o Students will be able to summarize univariate and bivariate data graphically and numerically.
- o Students will be able to use inference procedures to draw conclusions about parameters.
- o Students will understand how the properties of probability are used in the inference procedures of hypothesis testing and confidence intervals.
- o Students will be able to use calculus to understand and analyze the concepts in the course.

SCOPE AND SEQUENCE:

UNIT	TARGETED SKILLS
Statistical Thinking	<ul style="list-style-type: none"> o Recognize variation in everyday situations o Understand how variation impacts decisions o Begin thinking about likelihood of events o Design, implement, and analyze sampling strategies.
Probability	<ul style="list-style-type: none"> o Use the terminology associated with probability o Use addition and multiplication rules o Work with conditional probability o Use tree diagrams to illustrate situations o *Understand and complete actuary problems
Random Variables	<ul style="list-style-type: none"> o Use the theoretical definitions of expected value and variance to calculate and analyze random variables. o Derive rules for transforming random variables. o Visualize probability distributions with appropriate graphs. o Analyze specific discrete distributions: Binomial and Geometric o Analyze specific continuous distributions: Uniform, Normal, Chi square o
1 Proportion Inference	<ul style="list-style-type: none"> o Design simulations for 1 proportion situations o Use a structured approach to conducting an inference test o Understand the significance of a p-value o Understand the significance level of a test o Create a confidence interval for a single proportion o Interpret a confidence interval for a single proportion
2 Proportion Inference	<ul style="list-style-type: none"> o Understand the sampling distribution of the difference of proportions o Conduct hypothesis tests for 2 proportions o Create confidence intervals for 2 proportions o Use JMP for inference



Categorical Inferences	<ul style="list-style-type: none">○ Understand the characteristics of the chi square distribution○ Conduct goodness of fit test○ Conduct independence test○ Conduct homogeneity test
Analyzing Quantitative Data	<ul style="list-style-type: none">○ Create and analyze dot plots, stem and leaf plots, histograms, ogives, boxplots○ Calculate and interpret measures of center○ Calculate and interpret measures of spread○ Understand the central limit theorem
1 Sample Mean Inference	<ul style="list-style-type: none">○ Conduct a hypothesis test for 1 sample mean data○ Construct and interpret a confidence interval for a single mean
2 Sample Mean Inference	<ul style="list-style-type: none">○ Conduct a hypothesis test for 2 sample mean data○ Construct and interpret a confidence interval for a 2 means
Bivariate Data	<ul style="list-style-type: none">○ Construct and analyze graphs for bivariate data○ Construct and interpret measures of bivariate data: correlation, coefficient of determination, LSRL, predictions○ Understand relationships in bivariate data: cause and effect, confounding○ Use methods for straightening non-linear data○ Conduct inference for the slope
Potential Additional Topics	<ul style="list-style-type: none">○ ANOVA for regression and comparing multiple means.○ Multiple regression methods.



Upper School Curriculum

Math 550: Advanced Calculus

This course emphasizes an integrated approach to learning Calculus. Students utilize skills developed earlier in their mathematical courses to address conceptual and applied topics related to Calculus including limits, derivatives and integrals, as well as some of their basic applications. Most students choose to sit for the Advanced Placement (AP) examination (Calculus AB).

GENERAL COURSE OBJECTIVES:

- Students will be able to analyze and graph functions covered during Precalculus, more specifically, using information gained from the first and second derivatives.
- Students will understand the notion of the limit of a function and how to determine its value.
- Students will be able to optimize a quantity in a given system, such as the cost of building a container or the number of units of a product to create, using the first and second derivatives.
- Students will be able to measure the rates of change of functions using differentiation techniques.
- Students will be able to apply integration techniques to a function's rate of change in order to calculate the net change in the function's value.

SCOPE AND SEQUENCE:

UNIT	TARGETED SKILLS
Limits and Derivatives	<ul style="list-style-type: none"> ○ Graphically interpret the slope of the tangent line as a limit. ○ Algebraically compute the value of a limits in some indeterminate forms. ○ Evaluate determined limit forms. ○ Calculate left- and right-hand limits and assess continuity. ○ Determine the existence of vertical and horizontal asymptotes. ○ Interpret the meaning of the derivative within a word problem, including correct units. ○ Sketch qualitative derivative and anti-derivative graphs. ○ Use L'Hospital's Rule to evaluate limits in indeterminate form.
Differentiation Rules	<ul style="list-style-type: none"> ○ Compute derivatives using combinations of basic rules. ○ Apply the product, quotient and chain rules, including in combination. ○ Use the method of implicit differentiation to compute derivatives of nested and inverse functions.
Applications of Differentiation	<ul style="list-style-type: none"> ○ Solve word problems involving the concept of related rates and optimization. ○ Optimize a given function over an open or closed interval.
Integrals	<ul style="list-style-type: none"> ○ Interpret the definite integral as area, as well as a total change in the anti-derivative. ○ Find general anti-derivatives. ○ Apply the method of integration by substitution. ○ Use the fundamental theorem of calculus to show the inverse relationship between derivatives and anti-derivatives.
Areas and Volumes	<ul style="list-style-type: none"> ○ Compute areas of regions bounded by functions. ○ Compute volumes of solids by cross-section or of regions revolved about an axis. ○ Compute the average value of a function.



Differential Equations	<ul style="list-style-type: none">○ Solve separable differential equations analytically.○ Interpret a direction field to gain qualitative information about solutions to differential equations.
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Upper School Curriculum

Math 550: Advanced Calculus

This course covers the content of Calculus I ADV as well as Advanced Topics in Calculus. The pace and rigor of this course are intended for students with a commitment to learning advanced mathematics and a desire to pursue advanced mathematics in college. This course is structured using a modified blended model. In addition to regular coursework, students are expected to work during two additional designated periods to complete an online component of the course, requiring both consistent diligence and self-motivation. Most students choose to sit for the Advanced Placement (AP) examination (Calculus BC).

GENERAL COURSE OBJECTIVES:

- Students will be able to analyze and graph functions covered during Precalculus, more specifically, using information gained from the first and second derivatives.
- Students will understand the notion of the limit of a function and how to determine its value.
- Students will be able to optimize a quantity in a given system, such as the cost of building a container or the number of units of a product to create, using the first and second derivatives.
- Students will be able to measure the rates of change of functions using differentiation techniques.
- Students will be able to apply integration techniques to a function's rate of change in order to calculate the net change in the function's value.
- Students will be able to analyze and determine limit forms using more powerful techniques than were previously covered in Advanced Calculus.
- Students will be able to apply methods for analyzing differential equations to model and make predictions about population growth.
- Students will be able to apply integration techniques to solve physics and engineering-related problems such as calculating work or the concentration of an element being mixed into a solution.
- Students will be able to model and approximate functions using finite sums of polynomial terms.

SCOPE AND SEQUENCE:

UNIT	TARGETED SKILLS
Limits and Derivatives	Graphically interpret the slope of the tangent line as a limit. Algebraically compute the value of a limits in some indeterminate forms. Evaluate determined limit forms. Calculate left- and right-hand limits and assess continuity. Determine the existence of vertical and horizontal asymptotes. Interpret the meaning of the derivative within a word problem, including correct units. Sketch qualitative derivative and anti-derivative graphs.
Differentiation Rules	Compute derivatives using combinations of basic rules. Apply the product, quotient and chain rules, including in combination. Use the method of implicit differentiation to compute derivatives of nested and inverse functions.
Applications of Differentiation	Solve word problems involving the concept of related rates and optimization. Optimize a given function over an open or closed interval.



Integrals	Interpret the definite integral as area, as well as a total change in the anti-derivative. Find general anti-derivatives. Apply the method of integration by substitution. Use the fundamental theorem of calculus to show the inverse relationship between derivatives and anti-derivatives.
Areas and Volumes	Compute areas of regions bounded by functions. Compute volumes of solids by cross-section or of regions revolved about an axis. Compute the average value of a function.
Differential Equations	Solve separable differential equations analytically. Interpret a direction field to gain qualitative information about solutions to differential equations.
Parametrics	Produce a qualitative parametric graphs of simple functions by plotting key points and thinking about intermediate trends. Apply calculus methods such as finding the equation of a tangent line and distance traveled to parametric functions.
Indeterminate Limit Forms	Recognize and learn approaches to solve limits in various indeterminate forms. Know when and how to apply L'Hopital's Rule.
Integration	Apply the method of integration by parts, and distinguish from the substitution method. Apply the method, when appropriate, of Partial Fractions, using long division first when necessary. Recognize cases when various integration tricks such as trigonometric substitution should be used. Recognize improper integrals, and how to evaluate them or at least determine whether they converge or diverge.
Applications of Integration	Compute the arc length covered by a function over an interval. Calculate the work done in physics-based word problems involving a continuous vector force field.
Differential Equations	Understand the connection between population modeling and simple exponential growth and logistics growth. Solve the logistics population model analytically. Analyze differential equations numerically using Euler's Method. Analyze differential equations qualitatively/graphically using the first and second derivatives. Solve word problems involving mixing and interpret the results physically.
Sequences and Series	Compute the limit of a sequence, or explain why the sequence is divergent. Know the basic series tests, the conditions necessary to apply them, and how to use them to Determine convergence/divergence, finding the sum of the series when possible. Use a power series to represent a function, and how to find its interval of convergence. Use Taylor Series to represent and approximate any function. Estimate the error, when possible, involved in approximating a function with a truncated series.
Polar Integration	Graph basic polar functions using key points and by considering intermediate trends. Calculate the areas bounded by polar curves using polar integration.



Upper School Curriculum

Math 625: Advanced Calculus I and II

This course covers the content of Calculus I ADV as well as Advanced Topics in Calculus. The pace and rigor of this course are intended for students with a commitment to learning advanced mathematics and a desire to pursue advanced mathematics in college. This course is structured using a modified blended model. In addition to regular coursework, students are expected to work during two additional designated periods to complete an online component of the course, requiring both consistent diligence and self-motivation. Most students choose to sit for the Advanced Placement (AP) examination (Calculus BC).

GENERAL COURSE OBJECTIVES:

- Students will be able to analyze and graph functions covered during Precalculus, more specifically, using information gained from the first and second derivatives.
- Students will understand the notion of the limit of a function and how to determine its value.
- Students will be able to optimize a quantity in a given system, such as the cost of building a container or the number of units of a product to create, using the first and second derivatives.
- Students will be able to measure the rates of change of functions using differentiation techniques.
- Students will be able to apply integration techniques to a function's rate of change in order to calculate the net change in the function's value.
- Students will be able to analyze and determine limit forms using more powerful techniques than were previously covered in Advanced Calculus.
- Students will be able to apply methods for analyzing differential equations to model and make predictions about population growth.
- Students will be able to apply integration techniques to solve physics and engineering-related problems such as calculating work or the concentration of an element being mixed into a solution.
- Students will be able to model and approximate functions using finite sums of polynomial terms.

SCOPE AND SEQUENCE:

UNIT	TARGETED SKILLS
Limits and Derivatives	<ul style="list-style-type: none"> ○ Graphically interpret the slope of the tangent line as a limit. ○ Algebraically compute the value of a limits in some indeterminate forms. ○ Evaluate determined limit forms. ○ Calculate left- and right-hand limits and assess continuity. ○ Determine the existence of vertical and horizontal asymptotes. ○ Interpret the meaning of the derivative within a word problem, including correct units. ○ Sketch qualitative derivative and anti-derivative graphs.
Differentiation Rules	<ul style="list-style-type: none"> ○ Compute derivatives using combinations of basic rules. ○ Apply the product, quotient and chain rules, including in combination. ○ Use the method of implicit differentiation to compute derivatives of nested and inverse functions.
Applications of Differentiation	<ul style="list-style-type: none"> ○ Solve word problems involving the concept of related rates and optimization. ○ Optimize a given function over an open or closed interval.
Integrals	<ul style="list-style-type: none"> ○ Interpret the definite integral as area, as well as a total change in the anti-derivative. ○ Find general anti-derivatives. ○ Apply the method of integration by substitution. ○ Use the fundamental theorem of calculus to show the inverse relationship between



	derivatives and anti-derivatives.
Areas and Volumes	<ul style="list-style-type: none">○ Compute areas of regions bounded by functions.○ Compute volumes of solids by cross-section or of regions revolved about an axis.○ Compute the average value of a function.
Differential Equations	<ul style="list-style-type: none">○ Solve separable differential equations analytically.○ Interpret a direction field to gain qualitative information about solutions to differential equations.
Parametrics	<ul style="list-style-type: none">○ Produce a qualitative parametric graphs of simple functions by plotting key points and thinking about intermediate trends.○ Apply calculus methods such as finding the equation of a tangent line and distance traveled to parametric functions.
Indeterminate Limit Forms	<ul style="list-style-type: none">○ Recognize and learn approaches to solve limits in various indeterminate forms.○ Know when and how to apply L'Hopital's Rule.
Integration	<ul style="list-style-type: none">○ Apply the method of integration by parts, and distinguish from the substitution method.○ Apply the method, when appropriate, of Partial Fractions, using long division first when necessary.○ Recognize cases when various integration tricks such as trigonometric substitution should be used.○ Recognize improper integrals, and how to evaluate them or at least determine whether they converge or diverge.
Applications of Integration	<ul style="list-style-type: none">○ Compute the arc length covered by a function over an interval.○ Calculate the work done in physics-based word problems involving a continuous vector force field.
Differential Equations	<ul style="list-style-type: none">○ Understand the connection between population modeling and simple exponential growth and logistics growth.○ Solve the logistics population model analytically.○ Analyze differential equations numerically using Euler's Method.○ Analyze differential equations qualitatively/graphically using the first and second derivatives.○ Solve word problems involving mixing and interpret the results physically.
Sequences and Series	<ul style="list-style-type: none">○ Compute the limit of a sequence, or explain why the sequence is divergent.○ Know the basic series tests, the conditions necessary to apply them, and how to use them to Determine convergence/divergence, finding the sum of the series when possible.○ Use a power series to represent a function, and how to find its interval of convergence.○ Use Taylor Series to represent and approximate any function.○ Estimate the error, when possible, involved in approximating a function with a truncated series.



Polar Integration	<ul style="list-style-type: none">o Graph basic polar functions using key points and by considering intermediate trends.o Calculate the areas bounded by polar curves using polar integration.
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Upper School Curriculum

Math 700: Advanced Topics in Calculus

In this seminar, students will be asked to uncover the mathematics together with assistance from the instructor. There are two primary goals of the course. First, having completed a year of the Calculus, students will be asked to revisit topics from the first year with greater attention to depth of knowledge. In addition, students will examine new areas of the Calculus, including but not limited to, Taylor Series and more advanced applications of integration and differential equations. Most students in this course choose to sit for the Advanced Placement (AP) examination (Calculus BC).

GENERAL COURSE OBJECTIVES:

- Students will be able to analyze and determine limit forms using more powerful techniques than were previously covered in Advanced Calculus.
- Students will be able to apply methods for analyzing differential equations to model and make predictions about population growth.
- Students will be able to apply integration techniques to solve physics and engineering-related problems such as calculating work or the concentration of an element being mixed into a solution.
- Students will be able to model and approximate functions using finite sums of polynomial terms.

SCOPE AND SEQUENCE:

UNIT	TARGETED SKILLS
Parametrics	Produce a qualitative parametric graphs of simple functions by plotting key points and thinking about intermediate trends. Apply calculus methods such as finding the equation of a tangent line and distance traveled to parametric functions.
Indeterminate Limit Forms	Recognize and learn approaches to solve limits in various indeterminate forms. Know when and how to apply L'Hopital's Rule.
Integration	Apply the method of integration by parts, and distinguish from the substitution method. Apply the method, when appropriate, of Partial Fractions, using long division first when necessary. Recognize cases when various integration tricks such as trigonometric substitution should be used. Recognize improper integrals, and how to evaluate them or at least determine whether they converge or diverge.
Applications of Integration	Compute the arc length covered by a function over an interval. Calculate the work done in physics-based word problems involving a continuous vector force field.
Differential Equations	Understand the connection between population modeling and simple exponential growth and logistics growth. Solve the logistics population model analytically. Analyze differential equations numerically using Euler's Method. Analyze differential equations qualitatively/graphically using the first and second derivatives. Solve word problems involving mixing and interpret the results physically.



Sequences and Series	Compute the limit of a sequence, or explain why the sequence is divergent. Know the basic series tests, the conditions necessary to apply them, and how to use them to Determine convergence/divergence, finding the sum of the series when possible. Use a power series to represent a function, and how to find its interval of convergence. Use Taylor Series to represent and approximate any function. Estimate the error, when possible, involved in approximating a function with a truncated series.
Polar Integration	Graph basic polar functions using key points and by considering intermediate trends. Calculate the areas bounded by polar curves using polar integration.
Differential Equations	Solve separable differential equations analytically. Interpret a direction field to gain qualitative information about solutions to differential equations.



Upper School Curriculum

Math 750: Multivariable Calculus Differential Equations

This course will cover the content of semester courses in college-level Multivariable Calculus (1st half of the year) and Differential Equations (2nd half of the year) and be taught in a blended format. Each week, there will be a mixture of contact periods with the teacher and virtual periods during which to complete online assignments and participate in an online discussion forum. This forum will allow peers to collaborate with each other, with their teacher in a facilitator role.

GENERAL COURSE OBJECTIVES:

- o Students will demonstrate mastery of the college-level content in both halves of this course.
- o Students will collaborate effectively with their peers, under the guidance of their teacher.
- o Students will demonstrate independence and maturity in keeping up with weekly course deadlines and responsibilities.

SCOPE AND SEQUENCE:

MULTIVARIABLE CALCULUS:

UNIT	TARGETED SKILLS
Vectors	<ul style="list-style-type: none"> o Vectors in a Plane o Vectors in Space o Dot and Cross Products o Lines and Planes in Space o Surfaces in Space o Cylindrical and Spherical Coordinates
Vector-Valued Functions	<ul style="list-style-type: none"> o Differentiation and Integration o Velocity and Acceleration o Tangent and Normal Vectors o Arc Length and Curvature
Functions of Several Variables	<ul style="list-style-type: none"> o Limits and Continuity o Partial Derivatives o Differentials o The Chain Rule o Directional Derivatives and Gradients o Tangent Planes and Normal Lines o Optimization
Multiple Integration	<ul style="list-style-type: none"> o Iterated Integrals o Double Integrals and Volume o Polar Coordinates o Center of Mass/Moment of Inertia o Surface Area o Triple Integrals and Applications o Triple Integrals in Other Coordinate Systems
Vector Analysis	<ul style="list-style-type: none"> o Vector Fields o Line Integrals o Conservative Vector Fields and Independence of Path o Green's Theorem o Parametric Surfaces o Divergence Theorem o Stokes's Theorem



DIFFERENTIAL EQUATIONS:

Unit	Targeted Skills
First Order Differential Equations	<ul style="list-style-type: none">○ Conventions○ Basic DEs○ Geometric Methods○ Numerical Methods○ Linear ODEs○ Integrating Factors○ Complex Arithmetic○ Sinusoidal Functions○ Constant Coefficients○ Exponential Input○ Autonomous Equations○ Linear vs. Non-Linear
Second Order Constant Coefficient Linear Equations	<ul style="list-style-type: none">○ Characteristic Equation○ Damped Oscillators○ Exponential Response○ Gain and Pule Lag○ Undetermined Coefficients○ Linear Operators○ Pure Resonance○ Frequency Response○ Applications